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03 BP 7210 Ouagadougou 03. Burkina Faso. Tél. : (+226) 25 33 16 36

Fax : *(+226 25 31 26 86)* – Email : *lecourrier@gmail.com,* Site web : *www.cedres.bf*

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SOMMAIRE

An Application of an Error Correction Model with Higher Order Cointegrated Variables to the Demand for Money

El Hadji GUEYE
elhadji.gueye@cesag.edu.sn

Abstract

This paper applies the maximum likelihood technique developed by *Krishnakumar J. and E.H. Gueye (1998)* to estimate the parameters of a money demand equation for Switzerland in which there are variable integrated of different orders and particularly of order greater than one (1). The procedure was implemented in MATLAB for estimating our empirical model. Our results turn out to be satisfactory with interesting economic interpretations. This confirms the relevance of the methodological approach considering the integration and cointegration of macroeconomic time series.

Keywords : Unit Root, Cointegration, Error Correction Model, Money Demand.

1. Introduction

Traditionally, a demand function for money is represented by :

$$M = f(Y, R, P) \qquad (1)$$

where M is the quantity of money demanded in nominal terms, Y is the real income, R is the rate of interest and P the price level, see for instance *Trehan (1988), Johansen and Juselius (1990), Hafer and Jansen (1991), Hendry and Ericsson (1991), Miller (1991), Ford (1995)* for recent studies on money demand taking into account non stationarity aspects. Let us briefly recall the expected effect of changes in Y, R and P on M according to basic economic theory.

The transaction and precautionary motives for holding money must be weighed against the opportunity cost expressed in terms of lost interests. Classical theory suggests that economic agents will keep holding money up to the point when the marginal benefit equals the marginal cost. The two motives mentioned for keeping money are generally represented by the level of income and prices, both positively influencing the demand for money. On the other hand, the interest rate is taken to represent the opportunity cost with a negative relationship postulated between this variable and cash holdings. It is also customary to assume exogeneity of the supply of money and hence look at the demand equation from a partial equilibrium point of view. We adopt the same approach. If the variables appearing in the relationship (1) are non-stationary, then the relationship is of relevance only if it represents a stationary combination. Otherwise we will be faced with a spurious regression problem, see *Phillips (1986)*.

The concepts of stationarity and cointegration are no longer new to econometricians and there is a vast number of articles covering a wide range of issues concerning the estimation of and inference in cointegrated systems. Initially this literature was confined to variables integrated of order one (1) starting from *Engle and Granger(1987)* and going on to *Stock (1987), Johansen (1988), Sims, Stock and Watson (1990), Phillips (1991)* and so on. Recently studies relating to systems with higher order integrated variables have emerged, see *Stock and Watson (1993), Kitamura (1995), Johansen (1992)* and *Johansen (1995)*. An important area of application of the statistical theory of higher order integration concerns the modelling of the monetary sector

.

The purpose of the study is to assess the effects of determinants of demand for money such as income, interest rates and the price level.The methodological approach using the technique developed by *Krishnakumar J. and E.H. Gueye (1998)*, consists in first testing the unit root hypotheses for the different variables in order to identify their order of integration, then we proceed to cointegration tests thus making it possible to set up the triangular representation followed by its transformation into error correction model. The maximum likelihood method will be used to estimate the parameters of this model.

Through this study, we will test the unit root hypotheses for the different variables to identify their order of integration and then proceed to cointegration tests. The results of the study will make it possible to verify that the monetary aggregate M1 is mainly used for the transactions of agents in particular the demand for money decreases less than proportionally to an increase of interest rates, also the effect of an increase of income on the demand for money is less than proportional and finally a rise of inflation results a more than proportional increase for the demand for money.

The study is organized as follows. In section 2, we examine the order of integration of the different series which is the first step before testing for possible cointegrations. Then we go on to various tests of cointegration among variables and combinations of variables integrated of the same order which in turn enable us to formulate a triangular representation of our empirical model in section 3: An error correction representation is derived from the triangular representation following the procedure described in a more theoretical paper by *Krishnakumar J. and E.H. Gueye (1998)* and estimated using the maximum likelihood method proposed in the same paper. Section 4 analyses the empirical results and draws some conclusions in section 5.

2. The Data

Our sample consists of seasonally adjusted quarterly data covering the period from the 1st quarter of 1969 to the 4th quarter of 1993. They were taken from the OECD publication *"Main Economic indicators"*. The four series used in the study are real GNP (for Y), Money supply in nominal terms (for M), implicit price deflator (for P) and discount rate (for R).

Keeping in mind that many observations are required to apply unit root tests, we chose the maximum possible periodicity given the availability of data concerning all the variables.

The graphs of the different series (in logarithms) and their first order and second order differences were plotted to get a preliminary idea as to the nature of their evolution over time. They are presented in Annex 1. From these figures, it can be concluded that all the variables are nonstationary, though it is difficult to predict exactly how many times they need to be differentiated to obtain stationarity. One point worth mentioning is that *lP, lY* and *lM* exhibit a similar pattern of evolution that is different from that of *lR*. This may lead us to believe that *lP, lY* and *lM* are integrated of the same order. However, we will see later that the unit root tests will confirm the order of integration of these variables.

Applying the augmented Dickey-Fuller tests, see *Dickey-Fuller (1979)* on regressions including a shift term (with no trend term) and selecting an optimal lag using *Akaike Information Criterion,* we obtain the following results :

Table 1

Variable	$\hat{\alpha}$	$t_{\hat{\alpha}}$	$P-value$	$optimal\ lag$	AIC
lP	0.99	-2.37	0.15	5	-6,47
$\triangle\ lP$	0.59	-2.51	0.112	4	-6.43
$\triangle^2\ lP$	-1.95	-5.73	$6.6\ 10^{-7}$	4	-6.35

Table 2

Variable	$\hat{\alpha}$	$t_{\hat{\alpha}}$	$P-value$	$optimal\ lag$	AIC
lY	0.99	-1.26	0.64	6	-5,45
$\triangle\ lY$	0.58	-2.25	0.19	5	-5,46
$\triangle^2\ lY$	-2.87	-6.63	$5.6\ 10^{-9}$	4	-5.423

Table 3

Variable	$\hat{\alpha}$	$t_{\hat{\alpha}}$	$P-value$	$optimal\ lag$	AIC
lR	0.915	-2.55	0.103	7	-1.213
$\triangle\ lR$	0.427	-4.418	0.00027	2	-1.23
$\triangle^2\ lR$	-1.484	-5.089	0.000014	5	-1.061

Table 4

Variable	$\hat{\alpha}$	$t_{\hat{\alpha}}$	$P-value$	$optimal\ lag$	AIC
lM	0.97	-1.93	0.32	8	-3.77
$\triangle\ lM$	0.067	-3.23	0.018	7	-3.75
$\triangle^2\ lM$	-4.69	-4.64	0.000109	10	-3.66

From Tables 1, 2, 3 and 4, we conclude that $lP{\sim}I(2)$, $lY{\sim}I(2)$, $lR{\sim}I(1)$ and $lM{\sim}I(1)$. This is in fact more or less agreement with the intuitive expectations that one might have looking at the graphs showing the evolution of these series, except in the case of Money supply (M). Barring this, the difference between the $I(2)$ and $I(1)$ variables could already be seen in the graphs.

Having established the order of integration of our variables, we proceed to conduct various cointegration tests following *Engle and Granger* (*1987*). For this purpose, variables of the same order are combined and tested. More specifically, the two $I(2)$ variables can be tested cointegration into an $I(2)$ or an $I(0)$ variable.

Then different combinations of the two $I(1)$variables and the differences of the two $I(2)$ variables can be tested for stationarity. In case the two $I(2)$ variables are found to cointegrate into an $I(1)$ variable, this combination can also be added to the list of $I(1)$ variables. Table 5 gives the main results used in the formulation of our model.

Table 5

Equations	$\hat{\alpha}$	$t_{\hat{\alpha}}$	$P-value$	$optimal\ lag$	AIC
(lY, lP)	0.968	-1.375	0.594	4	-5.599
(lM, lR)	0.905	-2.695	0.075	7	0.873
$(\triangle lY, \triangle lP)$	0.299	-3.751	0.0034	3	-5.599
$(lM, lR, (lY, lP), \triangle lP)$	0.875	-3.038	0.0314	7	0.97

These results can be interpreted as follows. The first line of Table 5 shows that lY and lP do not cointegrate to produce a stationary residual. Hence there is no $CI(2,2)$. However, they do combine to give an $I(1)$ variable as indicated by line 3 of this Table. Next, there is no cointegration between lM and lR only. Among the various possibilities combining $I(1)$ variables, we find that lM , lR the $I(1)$ combination between lY and lP and the first difference of lP all cointegrate to give stationary residuals, using lM as the normalized variable. This relationship is the crucial one from an economic point of view as it is in fact our equation of the demand for money.

3. The Empirical Model

Based on the findings of the last section, our model can be formulated as follows:

$$\begin{cases} \triangle^2 p_t = u_{1t} \\ \triangle r_t = u_{21t} \\ \triangle y_t = \theta_{12} \triangle p_t + u_{22t} \\ m_t = \theta_{21} r_t + \theta_{22} y_t + \theta_3 p_t + \theta_4 \triangle p_t + u_{3t} \end{cases} \quad (3)$$

With $p = lP$, $r = lR$, $y = lY$, $m = lM$.

Introducing the orthogonalization of *Stock and Watson (1993)*, we obtain

$\triangle^2 p_t = v_{1t}$

$\triangle r_t = D_{11}(L) \triangle^2 p_t + v_{21t}$

(4)

$\triangle y_t = D_{12}(L) \triangle^2 p_t + \theta_{12} \triangle p_t + v_{22t}$

$$m_t = D_2(L) \triangle^2 p_t + D_{31}(L) \triangle r_t + D_{32}(L)(\triangle y_t - \theta_{12} \triangle p_t) + \theta_{21} r_t + \theta_{22} y_t + \theta_3 p_t + \theta_4 \triangle p_t + v_{3t}$$

This system is then transformed into an error correction model along the lines of *Krishnakumar J. and E.H. Gueye (1998)* to give:

$$\triangle^2 z_t = A\xi_{t-1} + B \triangle z_{t-1} + C z_{t-2} + w_t \quad (5)$$

with $\xi'_{t-1}(1 \times 4p) = [\triangle^2 p_{t-\tau} \triangle^2 r_{t-\tau} \triangle^2 y_{t-\tau} \triangle^2 m_{t-\tau}]_{\tau=1,2,\ldots,p}$

$\triangle z'_{t-1}(1 \times 4) = (\triangle p_{t-1} , \triangle r_{t-1} , \triangle y_{t-1} , \triangle m_{t-1})$

$z'_{t-2}(1 \times 4) = (p_{t-2}, r_{t-2}, y_{t-2}, m_{t-2})$

$A(4 \times 4p) = [A_1 \; A_2 \; \ldots. \; A_{4p}]$

$$B(4 \times 4) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ \theta_{12} & 0 & -1 & 0 \\ \delta_1 & \delta_{21} & \delta_{22} & -2 \end{pmatrix}$$

$$C(4 \times 4) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \theta_3 & \theta_{21} & \theta_{22} & -2 \end{pmatrix}$$

where

1. $\delta_1 = [d_{30}^{**} - D_3(1)]\theta_1 + \theta_2\theta_1 + 2\theta_3 + \theta_4$
2. $\theta_1 = \begin{pmatrix} 0 \\ \theta_{12} \end{pmatrix}$
3. $\theta_2 = (\theta_{21} \;\; \theta_{22})$

Estimates of all the parameters of the model can be derived in a straightforward manner from the coefficient estimates of the error correction model except for θ_4 which can obtain as:

$$\theta_4 = \delta_1 + (\delta_{22} - 2\theta_{22})\theta_{12} - 2\theta_3$$

Pulling all the T observations, equation (5) can be written as:

$$\triangle^2 Z = A\xi + B \triangle Z_{-1} + CZ_{-2} + W' \tag{6}$$

where $\triangle^2 Z'(4 \times T) = [\triangle^2 z_1 \;\; \triangle^2 z_2 \; .... \; \triangle^2 z_T]$;
$\xi'_{-1}(4p \times T) = [\xi_0 \;\; \xi_1 \; .... \; \xi_{T-1}]$

$\triangle Z'_{-1}(4 \times T) = [\triangle z_0 \;\; \triangle z_1 \; .... \; \triangle z_{T-1}]$;
$Z'_{-2}(4 \times T) = [z_{-1} \;\; z_0 \; ..... \; z_{T-2}]$

$W'(4 \times T) = [w_1 \;\; w_2 \; .... \; w_T]$

Writing the above in vec form, we have:

$$vec(\triangle^2 Z) = (\xi \otimes I_n)vec(A) + (\triangle Z_{-1} \otimes I_n)vec(B) + (Z_{-2} \otimes I_n)vec(C) + vec(W') \tag{7}$$

where

$$vec(B)=\begin{pmatrix}0\\0\\\theta_{12}\\\delta_1\\0\\-1\\\delta_{21}\\0\\0\\-1\\\delta_{22}\\0\\0\\0\\0\\-2\end{pmatrix}=\begin{pmatrix}0\\0\\0\\0\\0\\-1\\0\\0\\0\\-1\\0\\0\\0\\0\\0\\-2\end{pmatrix}+\begin{pmatrix}0&0&0&0\\0&0&0&0\\1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&1\\0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\begin{pmatrix}\theta_{12}\\\delta_1\\\delta_{21}\\\delta_{22}\end{pmatrix}=q_1+R_1\Pi_1^*$$

$$vec(C)=\begin{pmatrix}0\\0\\0\\\theta_3\\0\\0\\0\\\theta_{21}\\0\\0\\0\\\theta_{22}\\0\\0\\0\\-1\end{pmatrix}=\begin{pmatrix}0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\-1\end{pmatrix}+\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\\1&0&0\\0&0&0\\0&0&0\\0&0&0\\0&1&0\\0&0&0\\0&0&0\\0&0&0\\0&0&1\\0&0&0\\0&0&0\\0&0&0\\0&0&0\end{pmatrix}\begin{pmatrix}\theta_3\\\theta_{21}\\\theta_{22}\end{pmatrix}=q_2+R_2\Pi_2$$

Note that the cointegrating relationship between lY and lP leads to the constraint

$$\theta_3 = -\theta_{22} \times \theta_{12}$$

Thus, only the last three elements of Π_1^* need to be estimated and θ_{12} can be derived from Π_1. For calculation purposes, Π_1^* without θ_{12} is named Π_1. That is $\Pi_1 = (\delta_1 , \delta_{21} , \delta_{22})'$.

We then build on the procedure followed for the estimation of the model by maximum likelihood described in *Krishnakumar J. and E.H. Gueye (1998)*. Essentially, it consists in first deriving an estimator of *A* given *B* and *C* and then substituting it in the equation and applying maximum likelihood to obtain estimators of *B*, *C* and **Ω**. We can then go back to the first step for calculating the estimator for *A*. We perform these estimations for various versions depending on the choice of elements to be included in ξ_{t-1} .

We will only consider $p = 1$ (as higher lags do not have significant coefficient. This leaves us with 4 lagged variables $\triangle^2 p_{t-1}$, $\triangle^2 r_{t-1}$, $\triangle^2 y_{t-1}$, $\triangle^2 m_{t-1}$ potentially appearing in all equations. However, once again all these variables do not turn out to be significant in each equation. All the possible scenarios were examined: $\xi_{t-1} = \triangle^2 p_{t-1}$; $\xi_{t-1} = \triangle^2 r_{t-1}$; $\xi_{t-1} =$ $(\triangle^2 p_{t-1} , \triangle^2 r_{t-1})'$; … ……,

$\xi_{t-1} = (\triangle^2 p_{t-1} , \triangle^2 r_{t-1} , \triangle^2 y_{t-1} , \triangle^2 m_{t-1})'$

Among these several error correction model scenarios, we have selected the best ones according to the criteria: (i) non negativeness of the variance, (ii) significance of coefficients and (iii) conformity of signs from an economic point of view. This selection rule leads us to finally retain the following four versions:

1. $\triangle^2 m_t = \alpha_4 \triangle^2 m_{t-1} + \delta_1 \triangle p_{t-1} + \delta_{21} \triangle r_{t-1} + \delta_{22} \triangle y_{t-1} - 2 \triangle m_{t-1} + \theta_3 p_{t-2} + \theta_{21} r_{t-2} + \theta_{22} y_{t-2} - m_{t-2} + w_{3t}$

2. $\triangle^2 m_t = \alpha_2 \triangle^2 r_{t-1} + \alpha_4 \triangle^2 m_{t-1} + \delta_1 \triangle p_{t-1} + \delta_{21} \triangle r_{t-1} + \delta_{22} \triangle y_{t-1} - 2 \triangle m_{t-1} + \theta_3 p_{t-2} + \theta_{21} r_{t-2} + \theta_{22} y_{t-2} - m_{t-2} + w_{3t}$

3. $\triangle^2 m_t = \alpha_3 \triangle^2 y_{t-1} + \alpha_4 \triangle^2 m_{t-1} + \delta_1 \triangle p_{t-1} + \delta_{21} \triangle r_{t-1} + \delta_{22} \triangle y_{t-1} - 2 \triangle m_{t-1} + \theta_3 p_{t-2} + \theta_{21} r_{t-2} + \theta_{22} y_{t-2} - m_{t-2} + w_{3t}$

4. $\triangle^2 m_t = \alpha_2 \triangle^2 r_{t-1} + \alpha_3 \triangle^2 y_{t-1} + \alpha_4 \triangle^2 m_{t-1} + \delta_1 \triangle p_{t-1} + \delta_{21} \triangle r_{t-1} + \delta_{22} \triangle y_{t-1} - 2 \triangle m_{t-1} + \theta_3 p_{t-2} + \theta_{21} r_{t-2} + \theta_{22} y_{t-2} - m_{t-2} + w_{3t}$

Estimation of the above four versions by maximum likelihood gives us estimates of
$\Pi_1 = (\delta_1 , \ \delta_2 , \ \delta_{22})'$; $\Pi_2 = (\theta_3 , \theta_2 , \theta_{22})'$; $vec(A)$ and those of their asymptotic standard errors. The asymptotic standard error of θ_{12} is calculated in the following way.
We have

$$\theta_{12} = f(\gamma) = -\frac{\theta_3}{\theta_{22}}$$

where $\gamma = (\theta_3 , \theta_{22})'$.
Thus, by virtue of the continuity theorem, $\hat{\theta}_{12}$ can be estimated by

$$\hat{\theta}_{12} = f(\hat{\gamma}) = \frac{\hat{\theta}_3}{\hat{\theta}_{22}}$$

A first order approximation of $f(\hat{\gamma})$ is given by

$$f(\hat{\gamma}) \cong f(\gamma) + \frac{\partial f}{\partial \gamma'}(\hat{\gamma})(\hat{\gamma} - \gamma)$$

$$\frac{\partial f}{\partial \gamma'}(\hat{\gamma}) = \left(\frac{\partial f}{\partial \theta_3}(\hat{\gamma}),\ \frac{\partial f}{\partial \theta_{22}}(\hat{\gamma})\right) = \left(-\frac{1}{\hat{\theta}_{22}}\ ,\ \frac{\hat{\theta}_3}{\hat{\theta}_{22}^2}\right)$$

$$AsyV(\hat{\theta}_{12}) = AsyV(f(\hat{\gamma})) \cong \frac{\partial f}{\partial \gamma'}(\hat{\gamma})AsyV(\hat{\gamma})\frac{\partial f}{\partial \gamma}(\hat{\gamma})$$

$$\gamma = \begin{pmatrix} \theta_3 \\ \theta_{22} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \delta_1 \\ \delta_{21} \\ \delta_{22} \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \theta_3 \\ \theta_{21} \\ \theta_{22} \end{pmatrix}$$

$$\Rightarrow \gamma = J_1\Pi_1 + J_2\Pi_2 = J\Pi$$

where $J = [J_1 \quad J_2] = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

and $\Pi = \begin{pmatrix} \Pi_1 \\ \Pi_2 \end{pmatrix}$

Therefore $\quad \hat{\gamma} = J\hat{\Pi}$ and $AsyV(\hat{\Theta}) = JAsyV(\hat{\Pi})J'$

Finally $\quad AsyV(\hat{\theta}_{12}) \cong \frac{\partial f}{\partial \gamma'}(\hat{\gamma})\, JAsyV(\hat{\Pi})J'\, \frac{\partial f}{\partial \gamma}(\hat{\gamma})$

Estimators of θ_4 and its asymptotic variance are obtained as follows:

We have

$$\theta_4 = g(\Theta) = \delta_1 + (\delta_{22} - 2\theta_{22})\theta_{12} - 2\theta_3 = \delta_1 - \delta_{22}\frac{\theta_3}{\theta_{22}}$$

where $\Theta = (\delta_1 \ \ \delta_{22} \ \ \theta_3 \ \ \theta_{22})'$ and g a continuous and differentiable function. Therefore, by virtue of the continuity theorem, an estimator of $g(\Theta)$ is given by $g(\hat{\Theta})$ with

$$\hat{\Theta} = (\hat{\delta}_1 \quad \hat{\delta}_{22} \quad \hat{\theta}_3 \quad \hat{\theta}_{22})'$$

Thus

$$\hat{\theta}_4 = g(\widehat{\Theta}) = \hat{\delta}_1 - \hat{\delta}_{22}\frac{\hat{\theta}_3}{\hat{\theta}_{22}}$$

For the asymptotic variance of $\hat{\theta}_4$, we use the same approach as for $\hat{\theta}_{12}$.

$$g(\widehat{\Theta}) \cong g(\Theta) + \frac{\partial g}{\partial \Theta'}(\widehat{\Theta})(\widehat{\Theta} - \Theta)$$

where $\frac{\partial g}{\partial \Theta \prime}(\widehat{\Theta}) = \begin{pmatrix} \frac{\partial g}{\partial \delta_1} & \frac{\partial g}{\partial \delta_{22}} & \frac{\partial g}{\partial \theta_{22}} & \frac{\partial g}{\partial \theta_3} \end{pmatrix}(\widehat{\Theta})$

$$= \begin{pmatrix} 1 & -\frac{\hat{\theta}_3}{\hat{\theta}_{22}} & \hat{\delta}_{22}\frac{\hat{\theta}_3}{\hat{\theta}_{22}^2} & -\frac{\hat{\delta}_{22}}{\hat{\theta}_{22}} \end{pmatrix}$$

Therefore

$$AsyV(\hat{\theta}_4) = AsyV\left(g(\widehat{\Theta})\right) \cong \frac{\partial g}{\partial \Theta'}(\widehat{\Theta}) AsyV(\widehat{\Theta}) \frac{\partial g}{\partial \Theta}(\widehat{\Theta})$$

$$\Theta = \begin{pmatrix} \delta_1 \\ \delta_{22} \\ \theta_3 \\ \theta_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \delta_1 \\ \delta_{21} \\ \delta_{22} \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \theta_3 \\ \theta_{21} \\ \theta_{22} \end{pmatrix}$$

$$\Longrightarrow \Theta = L_1\Pi_1 + L_2\Pi_2 = L\Pi$$

where $\mathrm{L} = [\, L_1 \;\; L_2 \,] = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

It follows that $\widehat{\Theta} = L\widehat{\Pi}$ and $AsyV(\widehat{\Theta}) = LAsyV(\widehat{\Pi})L'$

$$\Rightarrow \text{AsyV}(\hat{\theta}_4) \cong \frac{\partial g}{\partial \Theta'}(\hat{\Theta})\, L\, AsyV(\hat{\Pi})\, L' \frac{\partial g}{\partial \Theta}(\hat{\Theta})$$

The estimation method was programmed in MATLAB and the code is available from the authors upon request.

4. Empirical Results

Table 1 of Annex 2 presents the result of maximum likelihood estimation of the four different versions of the error correction model, for the equation with $\Delta^2 m_t$ as the dependent variable.
Table 2 of Annex 2 gives the long-term relationships corresponding to these same versions, in which m_t is the normalized variable.

Before discussing the results, let us note that both $\Delta^2 m_t$ and Δm_t are stationary as m_t is $I(1)$
in our model. $\Delta^2 m_t$ as the dependent variable in our error correction model. It results from the transformation of the triangular representation. Hence the error correction model that simultaneously explains all the four dependent variables of our system can only be written for second order differenced variables. However, we can easily rewrite the equation for $\Delta^2 m_t$ as an equation for Δm_t by eliminating the term $-\Delta m_{t-1}$ from both sides of the equation.
The values given in brackets are the ratios of the estimated value estimated value over its estimated asymptotic standard error. Since asymptotic convergence has a normal distribution, the test for significance is based on the standard normal distribution.

Looking at Table 1 in Annex 2, we can say that the results are in general satisfactory. The coefficient estimators of $\Delta^2 m_{t-1}$, r_{t-2} and y_{t-2} are significant at the 5% level in all versions. If we allow for an error probability of 10 %, $\Delta^2 y_{t-1}$, Δp_{t-1} and p_{t-2} can be added to the list. It is to be noted that all versions produce a highly significant coefficient $\Delta^2 m_{t-1}$, the lagged endogenous variables, with a value greater than unity. All the coefficients of the integrated variables of order 1 and 2 are of the expected sign, except for that of Δr_{t-1} in version 1. More specifically, the coefficient of Δp_{t-1} is positive, Δr_{t-1} negative , Δy_{t-1} positive, p_{t-2} positive, r_{t-2} negative and y_{t-2} positive in the equation explaining the variations in money demand. Turning to the long term relationship (Table 2), once again we note that we have the right signs of coefficients and their significance at 5% level is in general accepted in all versions except for the coefficient of p_t which becomes significant at the 10 % level. Another important feature is that the values of the coefficients of the integrated variables are quite robust across the different versions indicating stability of coefficient in the underlying long-term relationships in which they appear.

Considering the signs and significance of the coefficients of the stationary variables too, we find that version 3 is the best of all. We rewrite below the short-run and the log-run equations of this version for the purpose of economic interpretation.

$$\Delta^2 m_t = -0.53\,\Delta^2 y_{t-1} + 2.18\,\Delta^2 m_{t-1} + 1.86\,\Delta p_{t-1} - 0.018\,\Delta r_{t-1} + 0.703\,\Delta y_{t-1}$$
$$-2\,\Delta m_{t-1} + 0.28 p_{t-2} - 0.17 r_{t-2} + 0.57 y_{t-2} - m_{t-2}$$

The long-run equation is represented by

$$m_t = -0.18 r_t + 0.57 y_t + 0.28 p_t + 1.51 \Delta p_{t-1}$$

From these equations, we can say the elasticity of money demand with respect the interest rate is – 0.18 and that with respect to income is 0.57. This means for every 1% growth there is an increase of 0.57% in money demand. Whereas for every 1% increase in interest rate, there is a decrease of 0.18% in money demand. Both these elasticities are less than unity. We also observe that an increase in inflation of 1% leads to an increase in 1.51% of demand for money and the elasticity is greater than unity. Our values of the elasticity with respect to interest rate compare well with those of other countries such as USA, UK. *Friedman and Schwartz (1982)* find that this elasticity is about -0.12 for these two countries.

Let us add a final remark regarding the price elasticity of money demand. Its value is about 0.3, well below unity. This may be surprising if we argue that economic agents do not have a monetary illusion i.e. they reason in terms of the real money that they would like to possess rather than in nominal terms. However, a value less than 1 for the price elasticity has also been found in other works, see *Preytrignet and Fischer (1991)*. A point worth mentioning here is that the two results are not strictly comparable as our price variables is I(2) whereas that of *Preytrignet and Fischer (1991)* is $I(1)$ and further, the definitions of M are not exactly the same in both studies. The research work carried out in this paper can be considered as a first step in the application of an estimation methodology incorporating higher order integrated variables and the author plan to undertake further investigations in the near future using alternative definitions of money, interest rate and prices. In fact, in the case of the variable y_t where the same definitions are used in both studies, the elasticities agree well with each other.

5. Conclusion

Integration and cointegration tests have shown that prices and money supply are integrated of order 2 while interest rates and money supply are integrated of order 1. It has also been shown that prices and income are cointegrated thus signifying the existence of a linear relationship between these two variables. The same is true between interest rates and money supply and between the change in prices reflecting inflation and the change in income. Thus these results made it possible to build the model on which the maximum likelihood method is applied for the estimation of the parameters.

Thus the results obtained showed that the monetary aggregate is mainly used for the transactions of agents in particular the demand for money decreases less than proportionally to an increase of interest rates, also the effect of an increase of income on the demand for money is less than proportional and finally a rise of inflation results a more than proportional increase for the demand for money.

The empirical analysis presented in our study is not exhaustive. This is particularly the case with the existence of a long-term relationship linking the monetary aggregate to the level of prices, to income, to the interest rate of the Swiss confederation. We believe that this application should be followed by a more detailed analysis of the presented model, in particular regarding the choice of the variables to be included in the error correction model.

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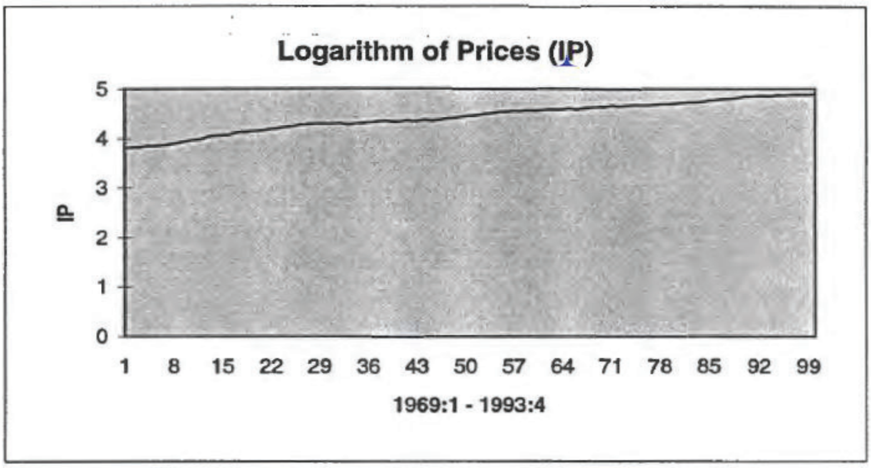
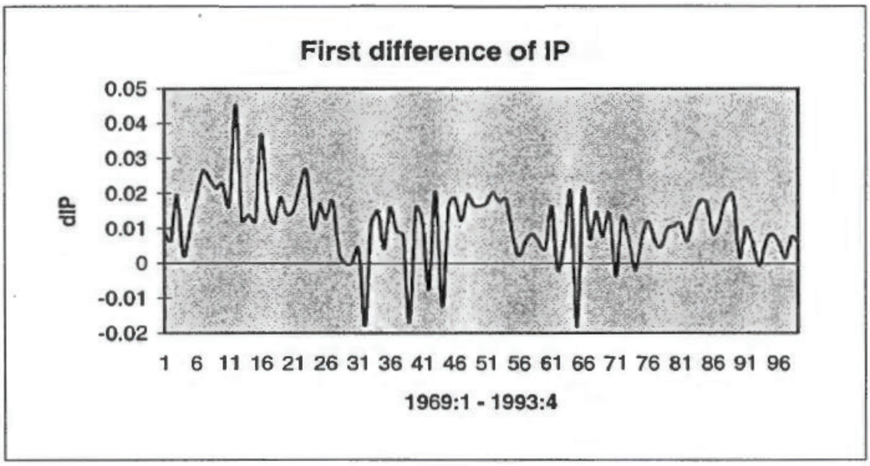
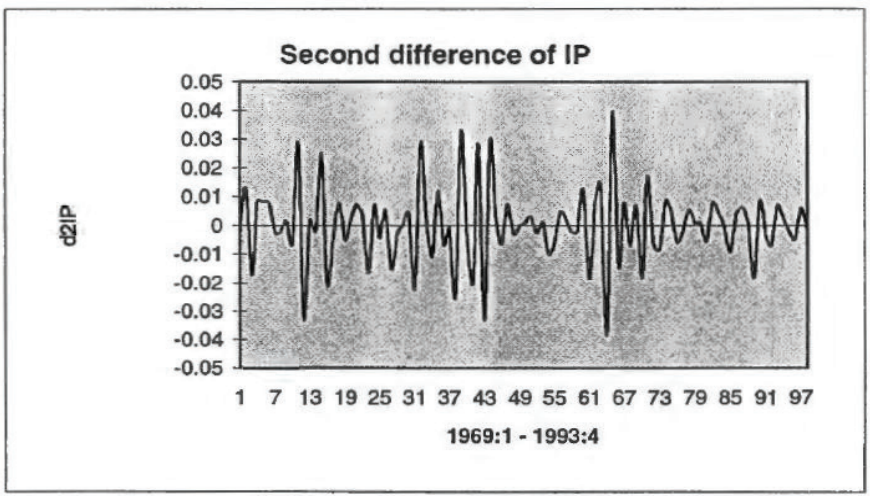
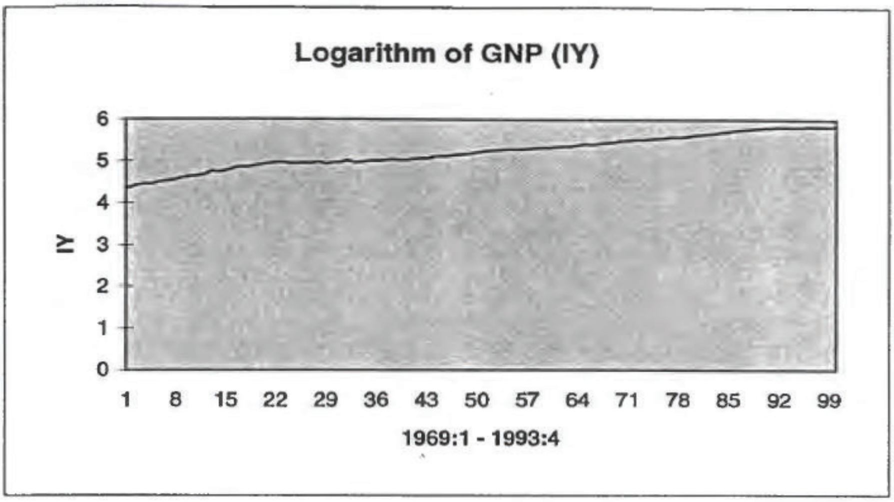
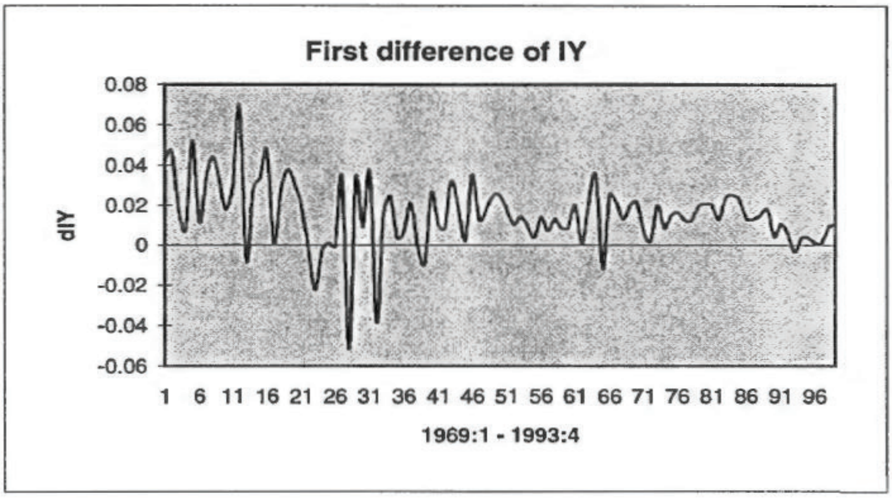
Annex 1: Graphs

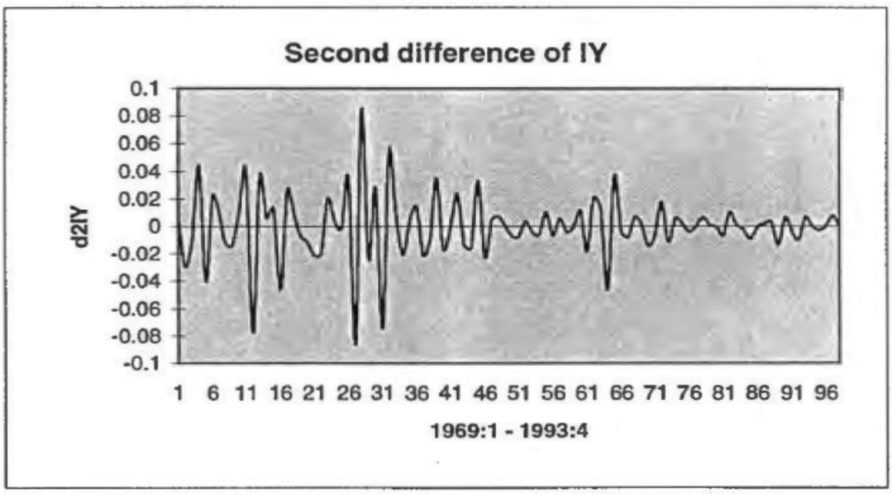


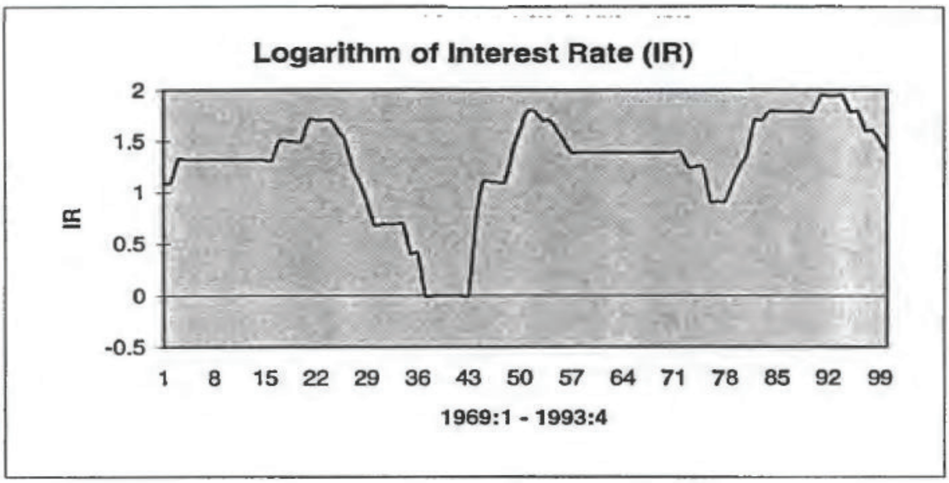


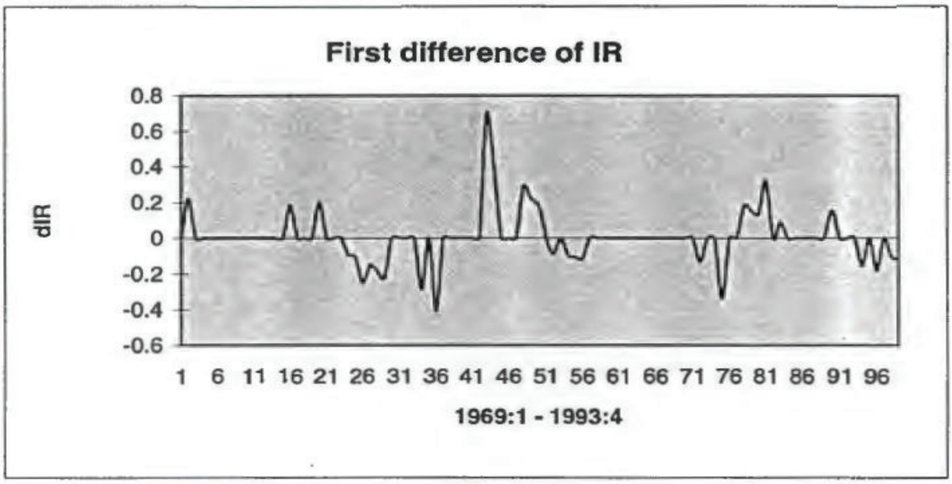


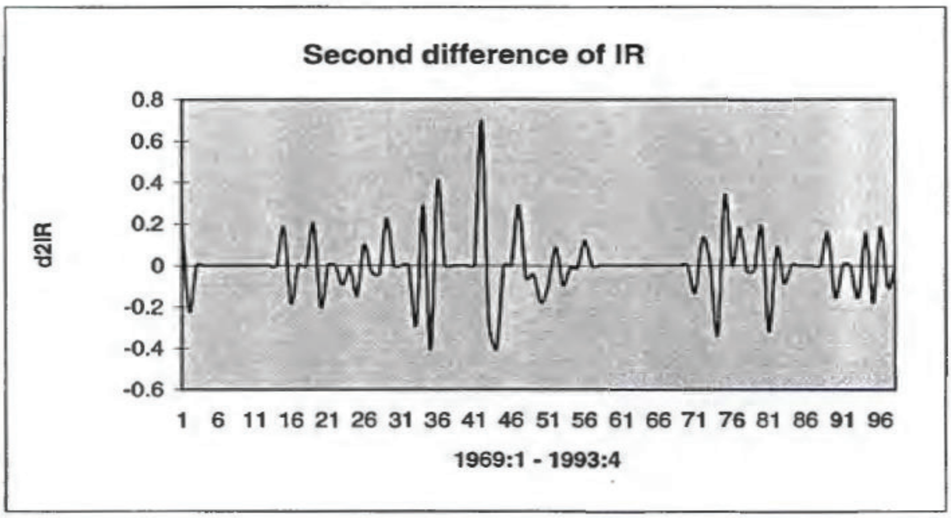

Logarithm of GNP (lY)
lY
6
5
4
3
2
1
0
1 8 15 22 29 36 43 50 57 64 71 78 85 92 99
1969:1 - 1993:4

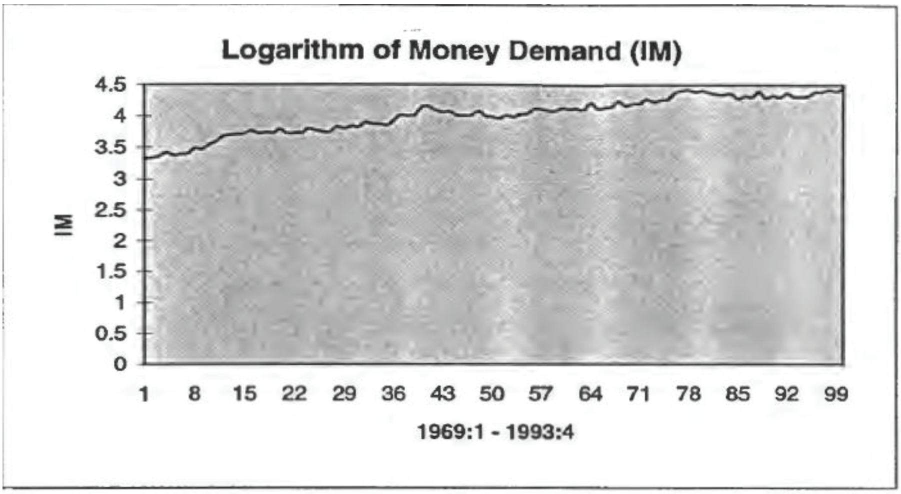

First difference of lY
dlY
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
1 6 11 16 21 26 31 36 41 46 51 56 61 66 71 76 81 86 91 96
1969:1 - 1993:4

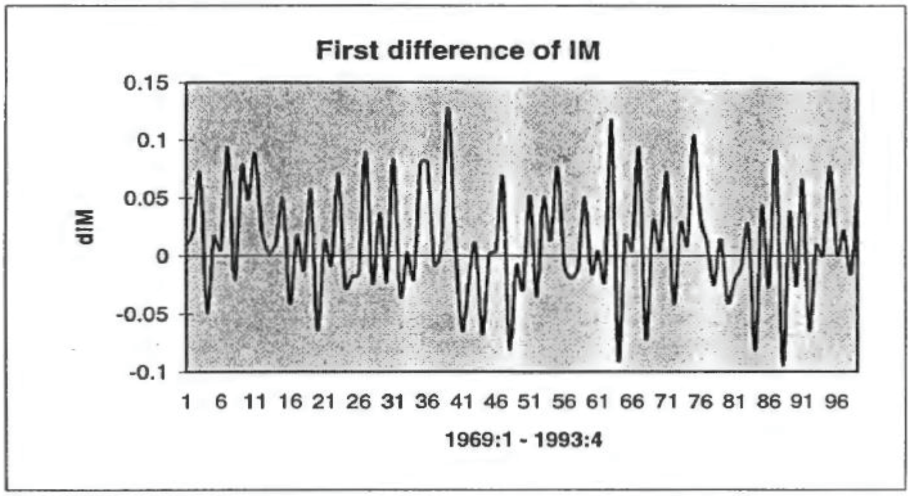

Second difference of lY
d2lY
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
1 6 11 16 21 26 31 36 41 46 51 56 61 66 71 76 81 86 91 96
1969:1 - 1993:4

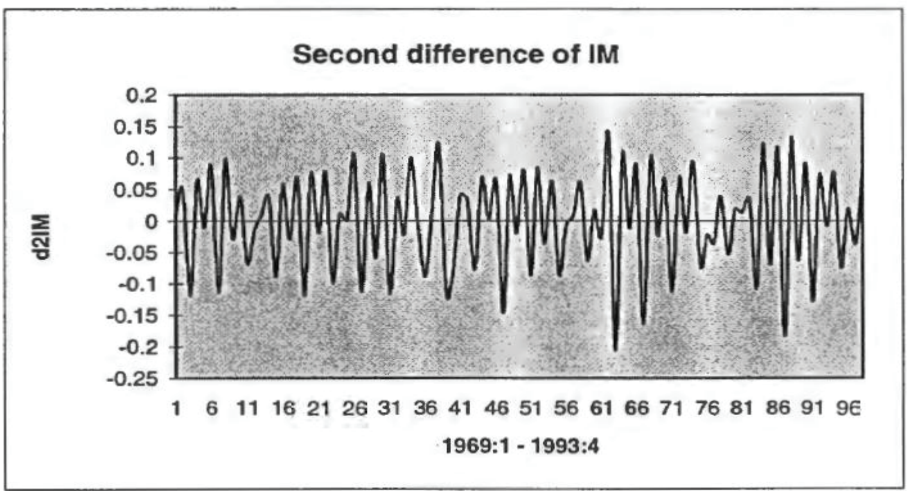

Logarithm of Interest Rate (IR)
IR
2
1.5
1
0.5
0
-0.5
1 8 15 22 29 36 43 50 57 64 71 78 85 92 99
1969:1 - 1993:4


First difference of IR
dIR
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
1 6 11 16 21 26 31 36 41 46 51 56 61 66 71 76 81 86 91 96
1969:1 - 1993:4


Second difference of IR
d2IR
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
1 6 11 16 21 26 31 36 41 46 51 56 61 66 71 76 81 86 91 96
1969:1 - 1993:4


Logarithm of Money Demand (IM)
IM
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
1 8 15 22 29 36 43 50 57 64 71 78 85 92 99
1969:1 - 1993:4


First difference of IM
dIM
0.15
0.1
0.05
0
-0.05
-0.1
1 6 11 16 21 26 31 36 41 46 51 56 61 66 71 76 81 86 91 96
1969:1 - 1993:4


Second difference of IM
d2IM
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
1 6 11 16 21 26 31 36 41 46 51 56 61 66 71 76 81 86 91 96
1969:1 - 1993:4

Annex 2 : Tables

Table 1: ***Error Correction Model****

Dependent variable: $\triangle^2 m_t$

$\triangle^2 m_t$	**Version 1**	**Version 2**	**Version 3**	**Version 4**
$\triangle^2 p_{t-1}$	----	----	----	----
$\triangle^2 r_{t-1}$	----	0.03102	----	0.0441
		(0.5826)		(0.8286)
$\triangle^2 y_{t-1}$	----	----	-0.5346	-0.5917
			(-1.5696)	(-1.7284)
$\triangle^2 m_{t-1}$	2.18015	2.1827	2.18706	2.1915
	(21.1165)	(21.0752)	(21.1846)	(21.1572)
$\triangle p_{t-1}$	1.9937	1.9732	1.8662	1.8235
	(1.8674)	(1.8487)	(1.7468)	(1.7075)
$\triangle r_{t-1}$	0.0005	-0.0202	-0.0183	-0.0499
	(0.0086)	(-0.2602)	(-0.277)	(-0.6128)
$\triangle y_{t-1}$	0.0923	0.0907	0.7036	0.7667
	(0.1511)	(0.1487)	(0.8401)	(0.9115)
$\triangle m_{t-1}$	-2	-2	-2	-2
p_{t-2}	0.3056	0.303	0.2844	0.2786
	(1.7719)	(1.7581)	(1.6478)	(1.6152)
r_{t-2}	-0.1738	-0.1719	-0.1768	-0.1745
	(-7.8405)	(-7.6383)	(-7.9562)	(-7.7671)
y_{t-2}	0.5526	0.5544	0.5698	0.5742
	(3.7118)	(3.7266)	(3.8262)	(3.8602)
m_{t-2}	-1	-1	-1	-1

* Figures inside brackets indicate t-ratio.

Table 2: ***Long-run relationship****

Normalized Variable: $\boldsymbol{m_t}$

$\boldsymbol{m_t}$	**Version 1**	**Version 2**	**Version 3**	**Version 4**
$\boldsymbol{r_t}$	-0.1738 (-7.8405)	-0.1719 (-7.6383)	-0.1768 (-7.9562)	-0.1745 (-7.7671)
$\boldsymbol{y_t}$	0.5526 (3.7118)	0.5544 (3.7266)	0.5698 (3.8262)	0.5742 (3.8602)
$\boldsymbol{p_t}$	0.3056 (1.7719)	0.303 (1.7581)	0.2844 (1.6478)	0.2786 (1.6152)
$\triangle \boldsymbol{p_t}$	1.9426 (1.5158)	1.9236 (1.5049)	1.5149 (1.1653)	1.4515 (1.1204)

* Figures inside brackets indicate t-ratio.